

Examples of Markov chains from Hopf algebras (in the sense of [Pan15a]). This version: January 8, 2015. If you spot an error, or know of any other Markov chains built in a similar way, please let me know.

| Markov chain                 | Hopf algebra / Hopf monoid                               | algebra is... |               |
|------------------------------|--|---------------|---------------|
|                              |  | commutative?  | cocommutative |
| shuffling                    | shuffle algebra $\mathcal{S}$                            | x             |               |
| inverse-shuffling            | free associative algebra $\mathcal{S}^*$                 |               | x             |
| edge-removal                 | $\mathcal{G}$  | x             | x             |
| edge-removal                 | $\mathcal{G}$  |               | x             |
| restriction-then-induction   | representations of symmetric groups                      | x             | x             |
| rock-breaking                | symmetric functions (partitions) $\subseteq \mathcal{G}$ | x             | x             |
| tree-pruning                 | Connes-Kreimer   | x             |               |
| descent-set-under-shuffling  | quasisymmetric functions                                 | x             |               |
| jeu-de-taquin                | Poirier-Reutenauer <b>FSym</b>                           |               |               |
| shuffle with standardisation | Malvenuto-Reutenauer <b>FQSym</b>                        |               |               |

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- [Pan13] C. Y. A. Pang. A Hopf-power Markov chain on compositions. In *25th International Conference on Formal Power Series and Algebraic Combinatorics*.
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- [Pan15b] C. Y. A. Pang. Lifting the down-up chain on partitions to permutations. *ArXiv e-prints*, August 2015.

2016. Curated by Amy Pang. Printer-friendly version, plus related summary tables, available at my website.

|   |       |         | basis                       | basis is...       |       |         |            | $ \mathcal{B}_1 $ | pro       |     |
|---|-------|---------|-----------------------------|-------------------|-------|---------|------------|-------------------|-----------|-----|
| ? | free? | cofree? |                             | free-commutative? | free? | cofree? | self-dual? | multigraded?      |           |     |
|   |       | x       | words / decks of cards      |                   |       | x       |            | x                 | arbitrary | sh  |
| x |       |         | words / decks of cards      |                   | x     |         |            | x                 | arbitrary | co  |
|   |       |         | unlabelled graphs           | x                 |       |         |            |                   | 1         | dis |
| x |       |         | labelled graphs             |                   | x     |         |            |                   | 1         | dis |
|   | x     |         | irreducible representations |                   |       |         | x          |                   | 1         | ex  |
|   | x     |         | elementary or complete      | x                 |       |         |            |                   | 1         | dis |
|   |       |         | rooted forests              | x                 |       |         |            |                   | 1         | dis |
|   | x     |         | fundamental (compositions)  |                   |       | x       |            |                   | 1         | (no |
| x |       |         | standard Young tableaux     |                   |       |         |            |                   | 1         | B2  |
| x | x     |         | fundamental (permutations)  |                   |       |         |            |                   | 1         | sh  |

REFER

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*Formal Power Series and Algebraic Combinatorics (FPSAC 2013)*, Discrete Math. Theor. Comput. Sci. Proc., AS, pages 499–510. Assoc. Discrete Math. Theor. Comp  
*Formal Power Series and Algebraic Combinatorics (FPSAC 2015)*, Discrete Math. Theor. Comput. Sci. Proc., AU, pages 49–60. Assoc

|  |                                       |                                    |  |           |
|--|---------------------------------------|------------------------------------|--|-----------|
| product                                | coproduct                             | rescaling                          | stationary distribution                          | reference |
| shuffle                                | deconcatenation                       | none                               | uniform  | [Pan13]   |
| deconcatenation                        | deshuffle                             | none                               | uniform  | [DPR15]   |
| joint union                            | induced on subsets                    | none                               | absorbing at empty graph                         | [DPR15]   |
| joint union                            | induced on subsets                    | none                               | absorbing at empty graph                         | [DPR15]   |
| internal induction                     | sum of restrictions                   | dimension                          | plancherel                                       | [Pan13]   |
| joint union                            | $\Delta((n)) = \sum(i) \otimes (n-i)$ | $\frac{n!}{\prod \lambda_i!}$      | absorbing at $(1, 1, \dots, 1)$                  | [DPR15]   |
| joint union                            | cut branches $\otimes$ trunks         | $\frac{n!}{\prod_{\text{desc}(v)}$ | absorbing at disconnected forest                 | [Pan13]   |
| non-explicit - use Projection Theorem) |                                       | none                               | proportion of permutations with this descent set | [Pan13]   |
| B2R: add outer box                     | B2R: unbump                           | dimension of shape                 | proportion of standard tableaux with this shape  | [Pan13]   |
| lifted shuffle                         | deconcatenate and standardise         | none                               | uniform  | [Pan13]   |

## REFERENCES

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| ences   |
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| 14, Sec. 6.1]   |
| R14, Sec. 6] [Pan14, Ex. 4.6.2, Ex. 4.7.2]  |
| R14, Ex. 3.1] [Pan14, Sec. 5.1]   |
| R14, Ex. 3.2]   |
| 14, Ex. 4.1.4, Ex. 4.3.2, Ex. 4.4.3, Ex. 4.5.3, Ex. 4.6.4] [Pan15a, Ex. 3.5] [Pan15b, Sec. 2] |
| R14, Sec. 4] [Pan14, Sec. 5.2]  |
| 14, Sec. 5.3] [Pan15a, Ex. 5.3]   |
| 13][Pan14, Sec. 6.2]  |
| 15b, Sec. 4]  |
| 15b, Sec. 5]  |