

Examples of Markov chains from Hopf algebras (in the sense of [Pan15a]). This version: January 8, 2016. Curated by Amy Pang. Printer-friendly version, plus related summary tables, available at my website.
 If you spot an error, or know of any other Markov chains built in a similar way, please let me know.

Markov chain	Hopf algebra / Hopf monoid	algebra is...				basis	basis is...					$ \mathcal{B}_1 $	product	coproduct	rescaling	stationary distribution	references
		commutative?	cocommutative?	free?	cofree?		free-commutative?	free?	cofree?	self-dual?	multigraded?						
shuffling	shuffle algebra \mathcal{S}	x			x	words / decks of cards			x		x	arbitrary	shuffle	deconcatenation	none	uniform	[Pan14, Sec. 6.1]
inverse-shuffling	free associative algebra \mathcal{S}^*		x	x		words / decks of cards		x			x	arbitrary	concatenation	deshuffle	none	uniform	[DPR14, Sec. 6] [Pan14, Ex. 4.6.2, Ex. 4.7.2]
edge-removal	\mathcal{G}	x	x			unlabelled graphs	x					1	disjoint union	induced on subsets	none	absorbing at empty graph	[DPR14, Ex. 3.1] [Pan14, Sec. 5.1]
edge-removal	\mathcal{G}		x	x		labelled graphs		x				1	disjoint union	induced on subsets	none	absorbing at empty graph	[DPR14, Ex. 3.2]
restriction-then-induction	representations of symmetric groups	x	x		x	irreducible representations				x		1	external induction	sum of restrictions	dimension	plancherel	[Pan14, Ex. 4.1.4, Ex. 4.3.2, Ex. 4.4.3, Ex. 4.5.3, Ex. 4.6.4] [Pan15a, Ex. 3.5] [Pan15b, Sec. 2]
rock-breaking	symmetric functions (partitions) $\subseteq \mathcal{G}$	x	x		x	elementary or complete	x					1	disjoint union	$\Delta((n)) = \sum(i) \otimes (n-i)$	$\frac{n!}{\prod \lambda_i!}$	absorbing at $(1, 1, \dots, 1)$	[DPR14, Sec. 4] [Pan14, Sec. 5.2]
tree-pruning	Connes-Kreimer	x				rooted forests	x					1	disjoint union	cut branches \otimes trunks	$\frac{n!}{\prod \text{desc}(v)}$	absorbing at disconnected forest	[Pan14, Sec. 5.3] [Pan15a, Ex. 5.3]
descent-set-under-shuffling	quasisymmetric functions	x			x	fundamental (compositions)			x			1	(non-explicit - use Projection Theorem)		none	proportion of permutations with this descent set	[Pan13][Pan14, Sec. 6.2]
jeu-de-taquin	Poirier-Reutenauer FSym			x		standard Young tableaux						1	B2R: add outer box	B2R: unbump	dimension of shape	proportion of standard tableaux with this shape	[Pan15b, Sec. 4]
shuffle with standardisation	Malvenuto-Reutenauer FQSym			x	x	fundamental (permutations)						1	shifted shuffle	deconcatenate and standardise	none	uniform	[Pan15b, Sec. 5]

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