Main examples of descent operators (convolution-of-projections maps) in the theory of Markov chains from Hopf
available at my website.

If you spot an error, or wish to add other maps to this list, please let me know.

Name of chain	Defining map (in all cases assume $\sum q_i = 1$)	Eigenvalues (i(λ) denotes
Hopf-square / Riffle-shuffle	$\frac{1}{2^n} m\Delta$	$2^{l(\lambda)-n}$
Hopf-power /a-handed shuffle	$\frac{1}{a^n} m^{[a]}\Delta^{[a]}$	$a^{l(\lambda)-n}$
biased Hopf-power / biased a-handed shuffle	$\sum q_1^{i_1} \dots q_a^{i_a} \operatorname{Proj}_{i_1} * \dots * \operatorname{Proj}_{i_a}$	the power su
ordered top-m-to-random	$\frac{1}{\binom{n}{m}} \operatorname{Proj}_m * \iota$	(doesn't simp
top-to-random (T2R)	$\frac{1}{n} \operatorname{Proj}_1 * \iota$	$\frac{1(\lambda)}{n}$
unordered top-m-to-random	$\frac{(n-m)!}{n!} \operatorname{Proj}_1^{*m} * \iota$	$\binom{1(\lambda)}{m}$
binomial top-to-random	$\sum_{m=0}^{n} \frac{1}{m!} q^m (1-q)^{n-m} \operatorname{Proj}_1^{*m} * \iota$	$(1-q)^{n-1(\lambda)}$
top-or-bottom-to-random (ToB2R)	$\frac{1}{n} (q \operatorname{Proj}_1 * \iota + (1-q)\iota * \operatorname{Proj}_1)$	$\frac{1(\lambda)}{n}$
trinomial top-and-bottom-to-random	$\sum_{m_1+m_2+m_3=n} \frac{1}{m_1!m_3!} q_1^{m_1} q_2^{m_2} q_3^{m_3} \operatorname{Proj}_1^{*m_1} * \iota * \operatorname{Proj}_1^{*m_3}$	$q_2^{n-1(\lambda)}$
top-and-bottom-to-random (T+B2R)	$\frac{1}{n(n-1)} (\operatorname{Proj}_1 * \iota * \operatorname{Proj}_1)$	$\frac{1(\lambda)(1(\lambda)-1)}{n(n-1)}$
top-m-and-bottom-m-to-random	$\frac{(n-2m)!}{n!} (\operatorname{Proj}_1^{*m} * \iota * \operatorname{Proj}_1^{*m})$	$\frac{1(\lambda)\dots(1(\lambda)-2n}{n\dots(n-2m)}$

References

[DFP92] P. Diaconis, J. A. Fill, and J. Pitman. Analysis of top to random shuffles. *Combin. Probab. Comput.*, 1(2):13

[Pan14] C. Y. A. Pang. Hopf algebras and Markov chains. *ArXiv e-prints*, December 2014. A revised thesis.

[Pan15] C. Y. A. Pang. Card-shuffling via convolutions of projections on combinatorial Hopf algebras. In *27th Int* Comput. Sci. Proc., ??, pages ??–?? Assoc. Discrete Math. Theor. Comput. Sci., Nancy, 2015. available on A

f algebras. This version: April 8, 2015. Curated by Amy Pang. Printer-friendly version, plus related summary tables,

β_λ	Eigenfunction formulae		References	
s number of parts of size i in λ)	cocommutative	commutative	[DFP92]	[Pan15]
	[Pan14, Th. 2.5.1.B]	[Pan14, Th. 2.5.1.A]		Sec. 1
	[Pan14, Th. 2.5.1.B]	[Pan14, Th. 2.5.1.A]		Sec. 1
m p_λ in the variables $q_1,\dots,q_a$				Ex. 3.2
plify nicely)			Sec. 2, Sec. 6 Ex.1	Ex. 3.3
	[Pan15, after Prop. 5.2]	forthcoming		Ex. 4.3
			Sec. 6 Ex.2	Ex. 3.3
)	[Pan15, Th. 5.1]	forthcoming	Sec. 2 Ex. 3	
			Sec. 6 Ex.4	Ex. 3.4, Ex. 4.4
			Sec. 6 Ex. 6	before Ex. 5.3
	forthcoming		Sec. 6 Ex. 5	
n)	forthcoming		Sec. 6 Ex. 3	

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Arxiv.