MATH 3407, Linear Algebra II
Semester 2, 2025
Class Information

Learning outcomes:

1. Repeat in abstract vector spaces (e.g. polynomials, matrices, functions), the calculations and reasoning about $\mathbb{R}^n$ from LinAl1.
2. Handle more complicated definitions, proofs, and theorems.
3. See connections between Linear Algebra and other math that you know*, see more general phrasings of theorems from LinAl1, feel qualitatively some of the principles of Linear Algebra and pure math.
4. Know about some Linear Algebra that interests you.

*The exact topics and theorems/proofs/examples will depend on who enrolls. I probably have enough material to teach for 20 weeks, and from that I will choose what I feel is most relevant to you as a group.

You are expected to be familiar with the course content of Math 2207 as taught between Semester 1 2022 and Semester 1 2023. (In future, we may assume Semester 2 2024 onwards.)

Assessment:

- **20% Homework** - each assignment is longer than in LinAl1, but last year I only gave 3 assignments.
- **40% 2 Tests** (to cover #1 and #2 above) - no minimum score, you can do badly on this and still pass. Last year, around 50% are "skill questions" (i.e. more than in LinAl1 final), but "skills" can look different in different vector spaces, and you may not have seen all skills in all spaces in homework and class.
- **40% Mini-Project** (to cover #4 above) - independently learn and write about any topic or application of Linear Algebra not covered in your previous coursework. You are graded on:
  - Knowledge and correctness of your chosen topic - how much do you understand of what you read, especially of the linear algebra involved;
  - Clarity and organisation of your explanation - are the mathematical statements accurately and precisely stated, is the work logically structured.

  After the essay submission, Dr. Pang will ask you questions to better assess the two criteria above.

  You should make your own small examples to check your understanding; beyond that, you are **NOT** expected to create anything original, you are **NOT** expected to code (unless you prefer coding your examples instead of hand-calculation), these do not score points. An excellent explanation of known mathematics can get a high grade.

  You are encouraged to use AI tools for brainstorming and to improve the language.

  You may work as a pair, but the grading is individual.

Math 2207 Linear Algebra I	Math 3407 Linear Algebra II
Vectors: $\mathbb{R}^n$; scalars: $\mathbb{R}$	Vectors: polynomials, matrices, functions; scalars: $\mathbb{R}$, $\mathbb{C}$,
Linear combinations linear systems, span, linear independence	 Span and linear independence of infinitely many vectors
Subspace basis from a spanning set, dimension orthogonal complement	 basis from a linearly independent set, combining subspaces other kinds of complements
Linear transformations standard matrix kernel and range rank+nullity=n by counting pivots	 multiple matrix representations, change of coordinates more subspaces related to transformations a statement comparing the kernel and range
Eigenvectors and diagonalisation factorising a matrix as $A = PDP^{-1}$	 expressing a linear transformation in the right bases, eigenvectors that span the space
Orthogonality dot product	 Quadratic forms and inner products
Vectors are printed in bold $\mathbf{v}$ or handwritten with vector sign $\vec{v}$	No vector sign, you should keep track what is a scalar, a vector, a function, a function of a function...
You are expected to prove: equations, linear independence	 equality of sets, equality of functions, directness of sums, etc.
Complete lecture slides available	Class is handwritten "live" **please alert me to notation inconsistencies**; poor-quality videos will be on Moodle after class, in case you cannot take complete notes

To save my writing time on the board, you are expected to be familiar with the shorthands:
$\therefore$ (therefore),
$\because$ (because),
$\forall$ (for all),
$\exists$ (there exists),
$\implies$ (implies),
$\Leftrightarrow$,"iff" (if and only if).