Reference sheet for
MATH 3407, Advanced Linear Algebra
Semester 2, 2019

More may be added throughout the semester.

Axioms of a vector space V over a field $\mathbb{F}$: (Chapter 6):

V1: There is an *addition* operation : $V \times V \to V$; given $\alpha, \beta \in V$, there is a vector $\alpha + \beta \in V$, called the *sum* of α and β.

V2: Addition is associative, i.e. $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ for all $\alpha, \beta, \gamma \in V$.

V3: Addition is commutative, i.e. $\alpha + \beta = \beta + \alpha$ for all $\alpha, \beta \in V$.

V4: V contains a *zero vector*, denoted by $\mathbf{0}$, satisfying $\alpha + \mathbf{0} = \alpha$ for all $\alpha \in V$.

V5: For each $\alpha \in V$, there is an element in V, denoted by $-\alpha$, such that $\alpha + (-\alpha) = \mathbf{0}$.

V6: There is a *scalar multiplication* operation : $\mathbb{F} \times V \to V$; given $a \in \mathbb{F}$, $\alpha \in V$, there is a vector $a\alpha \in V$.

V7: Scalar multiplication is associative, i.e. $a(b\alpha) = (ab)\alpha$ for all $a, b \in \mathbb{F}$, $\alpha \in V$.

V8: Scalar multiplication is distributive over addition, i.e. $a(\alpha + \beta) = a\alpha + a\beta$ for all $a \in \mathbb{F}$, $\alpha, \beta \in V$.

V9: Scalar multiplication satisfies $(a + b)\alpha = a\alpha + b\alpha$ for all $a, b \in \mathbb{F}$ and all $\alpha \in V$.

V10: Scalar multiplication satisfies $1\alpha = \alpha$, for all $\alpha \in V$. (Here, 1 denotes the unit of $\mathbb{F}$.)