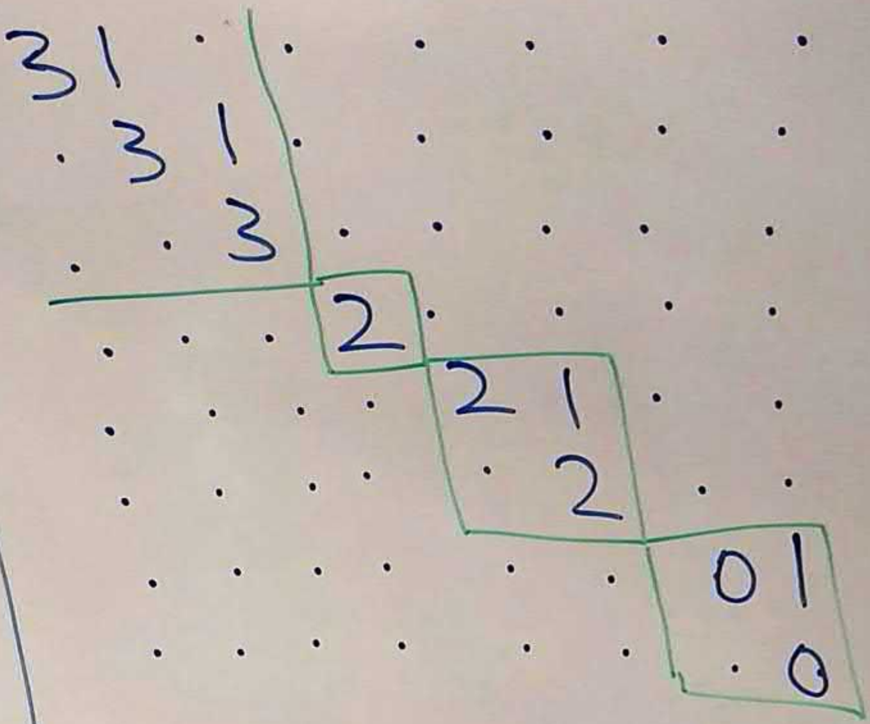
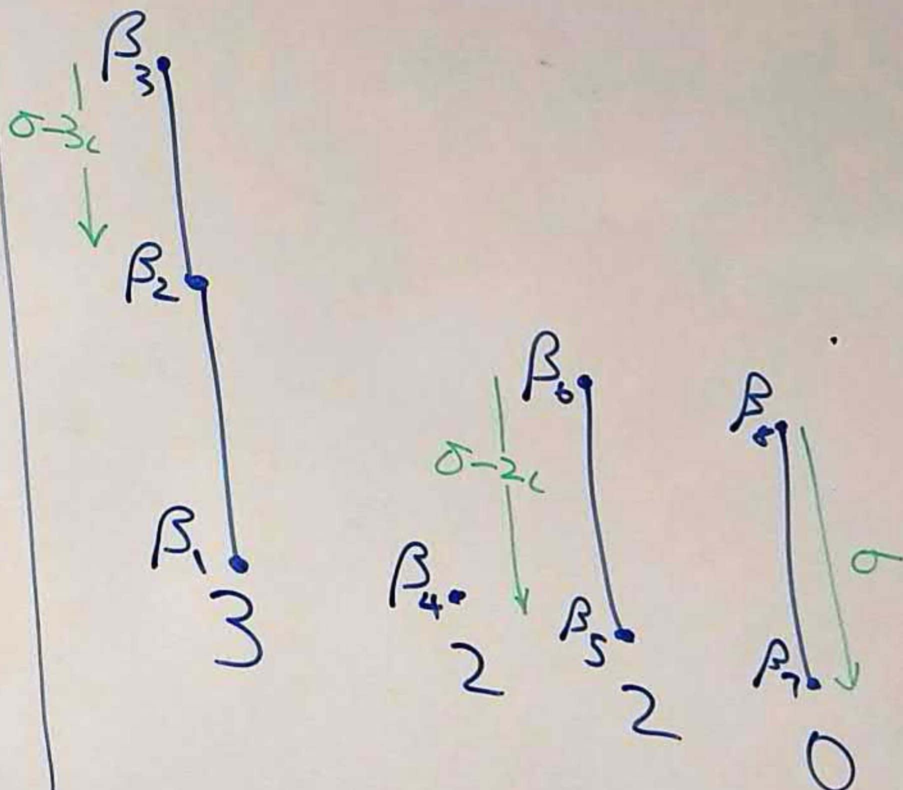
Ex: 8.2.6, to illustrate the page below





Last year: if J is diagonal, then column i of J is $\begin{pmatrix} 0 \\ \vdots \\ \lambda_i \\ \vdots \\ 0 \end{pmatrix} = [\sigma(\beta_i)]_{\mathcal{B}}$

i.e. $\sigma(\beta_i) = \lambda_i \beta_i$

Still true for size 1 block

e.g. in Ex. 8.2.6 $\sigma(\beta_4) = 2\beta_4$

In bigger blocks:

$\sigma(\beta_1) = 3\beta_1$ i.e. $(\sigma - 3\iota)\beta_1 = 0$

$\sigma(\beta_2) = 3\beta_2 + \beta_1$ $(\sigma - 3\iota)\beta_2 = \beta_1$

$\sigma(\beta_3) = 3\beta_3 + \beta_2$ $(\sigma - 3\iota)\beta_3 = \beta_2$

i.e. $(\sigma - \lambda\iota)$ sends a basis vector to the previous one.

In general, if $\{\beta_1, \dots, \beta_m\}$ is a basis for a λ-Jordan block,

then $(\sigma - \lambda\iota)\beta_1 = \vec{0}$ (i.e. β_1 is an eigenvector)

and, $\forall j > 1$, $(\sigma - \lambda\iota)\beta_j = \beta_{j-1}$

$(\sigma - \lambda\iota)^2 \beta_j = \beta_{j-2}$

$\vdots$

$(\sigma - \lambda\iota)^{j-1} \beta_j = \beta_1$

$(\sigma - \lambda\iota)^{j} \beta_j = \vec{0}$

$\therefore$ all β_j satisfy $(\sigma - \lambda\iota)^s(\beta_j) = \vec{0}$ for some s.

Def 8.2.1, 8.2.2, 8.2.4: Let $\sigma \in L(V,V)$

α is a generalised λ-eigenvector if $(\sigma - \lambda\iota)^s(\alpha) = \vec{0}$ for some s, and $\alpha \neq \vec{0}$.

$K_\lambda = \{\alpha \mid (\sigma - \lambda\iota)^s(\alpha) = \vec{0}$ for some $s\}$ is the generalised λ-eigenspace.

If $(\sigma - \lambda\iota)^s(\alpha) = \vec{0}$, but $(\sigma - \lambda\iota)^{s-1}(\alpha) \neq \vec{0}$

then the list (order is important)

$Z(\alpha; \lambda) = \{(\sigma - \lambda\iota)^{s-1}(\alpha), (\sigma - \lambda\iota)^{s-2}(\alpha), \dots (\sigma - \lambda\iota)\alpha, \alpha\}$

is a cycle of generalised λ-eigenvectors.

"λ-eigenstring"

$$Z(\alpha;\lambda) = \{\underbrace{(\sigma-\lambda_c)^{s-1}(\alpha)}_{\substack{\text{initial} \\ \text{vector} \\ \text{"bottom"}}}, \dots \; (\sigma-\lambda_c)(\alpha), \underbrace{\alpha}_{\substack{\text{end} \\ \text{vector} \\ \text{"top"}}}\}$$

$\therefore$ The basis $\mathcal{B}$ such that $[\sigma]_{\mathcal{B}}$ is in Jordan form (i.e. the columns of P such that $A = PJP^{-1}$) is a basis of eigenstrings (one string for each block).

Algorithm outline — Null Space Algorithm
(2nd algorithm in textbook)

0. Find eigenvalues by solving $\chi_\sigma(x) = 0$.
1. find the eigenstring diagram
 — i.e. find J.
2. find the longest eigenstrings
 — enough to find their tops, then apply $(\sigma - \lambda\iota)$
4. find the shorter eigenstrings.

be careful about linear independence.

How K_λ is like $\mathcal{E}_\lambda$:

Th. 8.2.3
- K_λ is a subspace
- $K_\lambda \supseteq \mathcal{E}_\lambda$
- K_λ is invariant under σ,
 i.e. $\sigma(K_\lambda) \subseteq K_\lambda$.

(see proof in book)